Reply by A.N.Ivanov and P. Kienle: Nowadays it is well established experimentally that neutrinos ν_α with lepton flavours $\alpha = e, \mu$ and τ are superpositions $|\nu_\alpha\rangle = \sum_j U^*_{\alpha j}|\nu_j\rangle$ of massive neutrino mass–eigenstates $|\nu_j\rangle$ with masses m_j, where $U^*_{\alpha j}$ are elements of the 3×3 unitary mixing matrix U, defined by mixing angles θ_{ij} [1]. The wave functions $|\nu_\alpha\rangle$ and $|\nu_j\rangle$ are orthogonal and used for the description of neutrino oscillations $\nu_\alpha \longleftrightarrow \nu_\beta$ with frequencies $\omega_{ij} = \Delta m^2_{ij}/2E$, where E is the neutrino energy and $\Delta m^2_{ij} = m^2_i - m^2_j$ [1]. In K–shell electron capture (EC) decays of the H–like heavy ions $m \to d + \nu_e$, where m and d are mother and daughter ions in their ground states [2, 3], one deals with an emission of electron neutrinos $|\nu_e\rangle = \sum_j U^*_{ej}|\nu_j\rangle$. Thus, the EC–decay rates of the H–like heavy ions are defined by the decay channels $m \to d_j + \nu_j$, where the final states are described by the orthogonal wave functions $\langle \nu_i d_i|d_j\nu_j\rangle = 0$ for $i \neq j$. The states of the daughter ions d_j differ in 3–momenta $\vec{q}_j$ and energies $E_d(\vec{q}_j)$. The massive neutrinos ν_j are produced with 3–momenta $\vec{k}_j = -\vec{q}_j$ and energies $E_j(\vec{k}_j)$, caused by conservation of energy and momentum in the decay channels $m \to d_j + \nu_j$. Since massive neutrinos ν_j are not detected they appear in the asymptotic states with 3–momenta $\vec{k}_j$, energies $E_j(\vec{k}_j)$ and energy differences $\omega_{ij} = \Delta m^2_{ij}/2M_m$, where M_m is the mass of the mother ion m [3]. In the GSI experiments [2, 3] the EC–decay channels $m \to d_j + \nu_j$ are measured by detecting the daughter ions d_j. If the daughter ions would be detected in the asymptotic states with 3–momenta $\vec{q}_j$ and energies $E_d(\vec{q}_j)$, the probability per unit time of the EC–decay $m \to d + \nu_e$ is equal to

$$P(m \to d\,\nu_e)(t) = \sum_j |U_{ej}|^2 P(m \to d_j\,\nu_j)(t) = \sum_j |U_{ej}|^2 \frac{d}{dt}|A(m \to d_j\nu_j)(t)|^2, \qquad (1)$$

where $A(m \to d_j\nu_j)(t)$ is the amplitude of the decay channel $m \to d_j + \nu_j$. However, this is not the case in the GSI experiments, where the time differential detection of the daughter ions with a time resolution τ_d leads to indistinguishability of daughter ions in the decay channels $m \to d_j + \nu_j$. As a result the daughter ions d_j are measured in the asymptotic state d with a 3–momentum $\vec{q}$ and an energy $E_d(\vec{q})$ such that $\vec{q} \simeq \vec{q}_j$ and $E_d(\vec{q}) \simeq E_d(\vec{q}_j)$ [3]. This does not violate the orthogonality of the wave functions in the final state $\langle \nu_i d|d\nu_j\rangle = 0$ for $i \neq j$. The energy and momentum uncertainties δE_d and $|\delta \vec{q}_d|$, respectively, induced by the time differential detection of the daughter ions, provide the overlap of the wave functions of the daughter ions if $\delta E_d \gg |\omega_{ij}|$ and $|\delta\vec{q}_d| \gg |\vec{q}_i - \vec{q}_j| = |\vec{k}_i - \vec{k}_j|$, where ω_{ij} present also the differences of the recoil energies of the daughter ions. The time differential detection of the daughter ions d_j in the asymptotic state with the 3–momentum $\vec{q}$ and an energy $E_d(\vec{q})$ results in a smearing of momenta and energies in the decay channels $m \to d_j + \nu_j$ around $\vec{q} + \vec{k}_j \simeq 0$ and $E_d(\vec{q}) + E_j(\vec{k}_j) \simeq M_m$. This is the origin of the non–vanishing interference terms in the probability per unit time of the EC–decay $m \to d + \nu_e$ [3]

$$P(m \to d\,\nu_e)(t) = \sum_j |U_{ej}|^2 P(m \to d\,\nu_j)(t) + 2\sum_{i>j} \frac{d}{dt}{\rm Re}[U^*_{ei}U_{ej}A^*(m \to d\,\nu_i)(t)A(m \to d\,\nu_j)(t)], \qquad (2)$$

where in comparison with Eq.(1) the second sum is caused by the interference terms. Since uncertainties δE_d and $|\delta\vec{q}_d|$ are rather small [3] and $|\vec{k}_j| = |\vec{q}_j| \simeq |\vec{q}| \simeq Q_{EC}$ and $Q_{EC} \gg m_j$, where Q_{EC} is the Q–value of the EC–decay $m \to d + \nu_e$, one can set the neutrino masses zero everywhere except for energy differences ω_{ij} and mixing angles θ_{ij} in the interference terms [3], and neutrino 3-momenta equal $\vec{k}_j = \vec{k}$. As a result the EC–decay rate is given by [3]

$$\lambda_{EC}(t) = \frac{1}{2M_m}\int P(m \to d\,\nu_e)(t)\,\frac{d^3q}{(2\pi)^3 2E_d}\,\frac{d^3k}{(2\pi)^3 2E_{\nu_e}} = \lambda_{EC}\Big(1 + 2\sum_{i>j}{\rm Re}[U^*_{ei}U_{ej}]\cos(\omega_{ij}t)\Big). \qquad (3)$$

Thus, the asymptotic orthogonality of the final state wave functions does not influence the observation of the time modulation of the EC–decays, observed in the GSI experiments with a time resolution $\tau_d \ll T_{ij}$ much shorter than the modulation periods $T_{ij} = 2\pi/\omega_{ij}$ [3]. This is unlike the assertion by Flambaum [4]. For $\tau_d \gg T_{ij}$ the daughter ions d_j become distinguishable with 3–momenta $\vec{q}_j$ and energies $E_d(\vec{q}_j)$ the time modulation vanishes (see Eq.(1)). For the theoretical description of the GSI data [2], accounting for the procedure for the detection of the daughter ions, one can use time–dependent perturbation theory and wave packets for the wave functions of the daughter ions, related to the density matrix description of unisolated quantum systems [5]. For technical details we refer to [3].

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